Appendix: Supporting Information for *Bureaucratic Resistance and Policy Inefficiency*

Contents

A Proofs

A.1 Proof for Proposition 1

Suppose bureaucrats never resist, so $b = 0$.

For an arbitrary threshold g' such that the voter reelects the reforming incumbent if and only if $g \geq g'$, the incumbent gets reelected with probability $1 - H(g')$ in $\omega = 0$ and $1 - H(g' - 1)$ in $\omega = 1$. Therefore, in $\omega = 0$, introducing reform ($a = 1$) is undominated if and only if

$$\rho + (1+\rho) \underbrace{[1 - H(g')]}_{\Pr[reelection|a=1,\omega=0]} \geq \frac{1}{2} \iff \rho \geq \rho_0(g') := \frac{H(g') - \frac{1}{2}}{2 - H(g')}. \tag{A1}$$

Notice that $\frac{1}{2} > \rho_0(g') \iff \frac{1}{2} > \frac{H(g')-\frac{1}{2}}{2-H(g')} \iff 2 - H(g') > 2H(g') - 1 \iff 1 > H(g')$.

If $\omega = 1$, $a = 1$ is undominated if and only if

$$\begin{aligned} \rho + (1+\rho) \underbrace{\Big([1 - H(g' - 1)]\Big)}_{\Pr[reelection|a=1,\omega=1,b=0]} &\geq \frac{1}{2} \\ \iff \rho \geq \rho_{B1}(g') := \frac{H(g'-1) - \frac{1}{2}}{2 - H(g'-1)}&. \end{aligned} \tag{A2}$$

Observe

$$\begin{aligned} E[\omega|g, g'] &= \frac{\Pr[\omega = 1]h(g|\omega = 1)\Pr[a = 1|\omega = 1, g']}{\Pr[\omega = 1]h(g|\omega = 1)\Pr[a = 1|\omega = 1, g'] + \Pr[\omega = 0]h(g|\omega = 0)\Pr[a = 1|\omega = 0, g']} \\ &= 1\Big/\left(1 + \frac{\Pr[\omega = 0]}{\Pr[\omega = 1]}\frac{h(g|\omega = 0)}{h(g|\omega = 1)}\frac{\Pr[a = 1|\omega = 0, g']}{\Pr[a = 1|\omega = 1, g']}\right). \end{aligned}$$

Define

$$I_B(g) = \frac{h(g)}{h(g-1)} \qquad R_B(g) = \frac{1 - \rho_0(g)}{1 - \rho_{B1}(g)}.$$

Then, for an arbitrary observation g and an arbitrary threshold g',

$$I_B(g)R_B(g') = \frac{\Pr[\omega = 0]}{\Pr[\omega = 1]}\frac{h(g|\omega = 0)}{h(g|\omega = 1)}\frac{\Pr[a = 1|\omega = 0]}{\Pr[a = 1|\omega = 1]}.$$

In equilibrium, this arbitrary threshold must be where the conditional expectation of ω

given the observed g is the same as the status quo's value:

$$E[\omega|g,g] = \frac{1}{1+I_B(g)R_B(g)} = q$$
$$\iff I_B(g)R_B(g) = \frac{1-q}{q}.$$

Let $g_B^*(q)$ denote such threshold. If $I_B(g)R_B(g)$ is monotonic with respect to g, then g_B^* is unique.

Lemma A1

$$I_B(g)R_B(g) = \frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)[2-H(g-1)]}{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)[2-H(g)]} = \frac{\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]}}{\frac{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)}{[2-H(g-1)]}}$$

is strictly decreasing in g.

Proof. When $h(g)$ is log-concave, $H(g)$ is also log-concave or its horizontal shifts. Also, log-concave functions are closed for multiplication. Thus, $\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]} > 0$ is log-concave. Notice that $\frac{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)}{[2-H(g-1)]}$ is a horizontal shift of $\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]} > 0$. Since a log-concave function satisfies the MLRP with respect to a horizontal shift (Saumard and Wellner, 2014), $\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]} / \frac{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)}{[2-H(g-1)]}$ is monotonic in g.

To see this in detail, observe that

$$\frac{\partial}{\partial g}\frac{\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]}}{\frac{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)}{[2-H(g-1)]}} \propto \frac{\partial}{\partial g}\log\frac{\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]}}{\frac{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)}{[2-H(g-1)]}}.$$

since all the components are positives: $h > 0$ and $H \in (0,1)$, so $1-H > 0$ and $2-H > 0$.

Notice

$$\log\frac{\frac{h(g)\left(2[1-H(g)]+\frac{1}{2}\right)}{[2-H(g)]}}{\frac{h(g-1)\left(2[1-H(g-1)]+\frac{1}{2}\right)}{[2-H(g-1)]}} = \log\frac{h(g)}{h(g-1)} + \log\frac{2[1-H(g)]+\frac{1}{2}}{2-H(g)} - \log\frac{2[1-H(g-1)]+\frac{1}{2}}{2-H(g-1)}.$$

First, we know that $\log\frac{h(g)}{h(g-1)}$ is strictly decreasing in g by the MLRP. Secondly, observe that

$$\frac{2[1-H(g)]+\frac{1}{2}}{2-H(g)} = \frac{2[2-H(g)]-2+\frac{1}{2}}{2-H(g)} = 2-\frac{\frac{3}{2}}{2-H(g)}.$$

Since $H(g)$ is increasing in g, $2-H(g)$ is decreasing in g, $\frac{\frac{3}{2}}{2-H(g)}$ is increasing, and, therefore, $2-\frac{\frac{3}{2}}{2-H(g)}$ is decreasing in g. The same applies for $H(g-1)$. Thus, $\log I_B(g) + \log R_B(g)$ is strictly decreasing in g, and therefore, $I_B(g)R_B(g)$ is strictly decreasing in g because log is strictly increasing. ■

Consequently, there exists a unique $g^*_B(q)$ such that $I_B(g^*_B(q))R_B(g^*_B(q)) = \frac{1-q}{q}$, by the Intermediate Value Theorem.

Let $\rho^*_{B0}(q) := \rho_0(g^*_B(q))$ and $\rho^*_{B1}(q) := \rho_{B1}(g^*_B(q))$.

$g^*_B(q)$ is increasing in q. To see why $g^*_B(q)$ is increasing in q, observe that, $I_B(g)R_B(g)$ is monotonically decreasing in g and $\frac{1-q}{q}$ is also monotonically decreasing in q; it takes a larger $g^*_B(q)$ that solves the equation $I_B(g)R_B(g) = \frac{1-q}{q}$ as q increases. To see this more directly, apply the Implicit Function Theorem.[1] First observe $I_B(g)R_B(g) - \frac{1-q}{q} = 0$. This implies that

$$\frac{\partial g^*_B(q)}{\partial q} = -\frac{\frac{\partial}{\partial q}\left[I_B(g)R_B(g) - \frac{1-q}{q}\right]}{\frac{\partial}{\partial g}\left[I_B(g)R_B(g) - \frac{1-q}{q}\right]}.$$

Since $\frac{\partial}{\partial q}\left[I_B(g)R_B(g) - \frac{1-q}{q}\right] = \frac{\partial}{\partial q}\left[-\frac{1-q}{q}\right] > 0$ and $\frac{\partial}{\partial g}\left[I_B(g)R_B(g) - \frac{1-q}{q}\right] = \frac{\partial}{\partial g}\left[I_B(g)R_B(g)\right] < 0$, so the fraction $\frac{\frac{\partial}{\partial q}\left[I_B(g)R_B(g) - \frac{1-q}{q}\right]}{\frac{\partial}{\partial g}\left[I_B(g)R_B(g) - \frac{1-q}{q}\right]} < 0$, and therefore, $\frac{\partial g^*_B(q)}{\partial q} > 0$.

Since $g^*_B(q)$ is increasing in q, $\rho_0(g)$ and $\rho_{B1}(g)$ are increasing in g, so both are increasing in q. For instance, $\frac{\mathrm{d}\rho_0(g^*_B(q))}{\mathrm{q}} = \frac{\partial \rho_0(g^*_B(q))}{\partial g^*_B(q)} \frac{\partial g^*_B(q)}{q} > 0$ since $\frac{\partial \rho_0(g^*_B(q))}{\partial g^*_B(q)} > 0$ and $\frac{\partial g^*_B(q)}{\partial q} > 0$.

In turn, $g^*_B(q)$ is invertible; there exists a function $\hat{q}(g) : \mathbb{R} \to (0,1)$ such that $\hat{q}(g^*_B(q)) = q \; \forall q \in (0,1)$.

A.2 Proof for Proposition 2

As in Equation (5), for an arbitrary threshold g', bureaucrats resist if and only if $\omega = 1, a = 1$, and

$$\kappa \geq \max\{\hat{\kappa}(g';c), 1\} \text{ such that } \hat{\kappa}(g';c) := \frac{c}{H(g') - H(g'-1)}.$$

It is useful that $H(g) - H(g-1)$ is single-peaked in g and attains a unique peak at $g = 1/2$: $h(g) - h(g-1) \geq 0 \iff g \leq 1/2$.

Since resistance occurs only when $\omega = 1$, the incumbent's decision given an arbitrary threshold g' is unaffected: $a = 1$ in $\omega = 0$ if and only if $\rho \geq \hat{\rho}_0(g')$.

Given the probability of resistance, $\hat{\kappa}(g';c)$, $a = 1$ in $\omega = 1$ is undominated for the incumbent if and only if

$$\rho + (1+\rho)\underbrace{\Big(\hat{\kappa}(g';c)[1 - H(g'-1)] + [1 - \hat{\kappa}(g';c)][1 - H(g')]\Big)}_{\Pr[reelection|a=1,\omega=1,b=0]} \geq \frac{1}{2}$$

$$\iff \rho + (1+\rho)\Big(\hat{\kappa}(g';c)[H(g') - H(g'-1)] + [1 - H(g')]\Big).$$

Since $\hat{\kappa}(g';c)[H(g') - H(g'-1)] = \frac{c}{H(g') - H(g'-1)}[H(g') - H(g'-1)] = c$ if $\hat{\kappa}(g';c) < 1$ and

[1] We thank Sebastian Hernandez for the suggestion.

$\hat{\kappa}(g';c)[H(g') - H(g'-1)] = [H(g') - H(g'-1)]$ if $\hat{\kappa} = 1$, define

$$\hat{c}(g';c) = \begin{cases} c & \text{if } \hat{\kappa}(g';c) < 1 \\ H(g') - H(g'-1) & \text{if } \hat{\kappa}(g';c) = 1, \end{cases}$$

$a = 1$ in $\omega = 1$ is undominated if and only if

$$\iff \rho + (1+\rho)\Big(\hat{c}(g') + [1 - H(g')]\Big) \geq \frac{1}{2} \iff \rho \geq \rho_1(g') := \frac{H(g') - \frac{1}{2} - \hat{c}(g')}{2 - H(g') + \hat{c}(g')}. \quad \text{(A3)}$$

Define

$$I(g,g') = \frac{h(g)}{h(g) + \hat{\kappa}(g';c)[h(g-1) - h(g)]} \qquad R(g') = \frac{1 - \rho_0(g')}{1 - \rho_1(g')}.$$

Then,

$$E[\omega|g,g'] = \frac{1}{1 + I(g,g')R(g')},$$

so the equilibrium threshold g^* must satisfies

$$E[\omega|g^*,g^*] = q \iff I(g^*,g^*)R(g^*) = \frac{1-q}{q}.$$

Lemma A2 *$I(g,g)R(g)$ is decreasing in g.*

Proof. Suppose $\hat{\kappa}(g) \geq 1$. Then, the proof for Proposition 1 implies that $I(g,g)R(g)$ is decreasing in g.

Suppose $\hat{\kappa}(g) < 1$. Take logarithm to get

$$\log I(g,g)R(g) = \log I(g,g) + \log R(g).$$

Since log is an increasing function, $I(g,g)R(g)$ is decreasing in g if $I(g,g)$ and $R(g)$ are decreasing in g independently.

$$I(g,g) = \frac{h(g)}{h(g) + c\frac{h(g-1)-h(g)}{H(g)-H(g-1)}} = \frac{1}{1 + c\frac{[h(g-1)/h(g)]-1}{H(g)-H(g-1)}}$$

is decreasing in g since

$$\varphi(g) := \frac{[h(g-1)/h(g)] - 1}{H(g) - H(g-1)}$$

is increasing in g:

$$\varphi'(g) = \frac{[H(g)-H(g-1)]\frac{\partial}{\partial g}\Big(\frac{h(g-1)}{h(g)}-1\Big) + \big(\frac{h(g-1)}{h(g)}-1\Big)\frac{\partial}{\partial g}[H(g)-H(g-1)]}{[H(g)-H(g-1)]^2} > 0$$
$$\iff [H(g)-H(g-1)]\frac{h'(g-1)h(g)-h'(g)h(g-1)}{\Big(h(g)\Big)^2} - [h(g)-h(g-1)]\Big(\frac{h(g-1)}{h(g)}-1\Big) > 0.$$

First, the first term, $[H(g)-H(g-1)]\frac{h'(g-1)h(g)-h'(g)h(g-1)}{[h(g)]^2}$ is positive. To see this, first notice that $[H(g)-H(g-1)] > 0$ due to the first-order stochastic dominance. Second, the log-concavity of h ensures $\frac{\partial}{\partial g}\frac{h'(g)}{h(g)} < 0 \iff h''(g)h(g) < [h'(g)]^2$ (Bagnoli and Bergstrom, 2006). Therefore, $h'(g-1)h(g) - h'(g)h(g-1) > 0 \iff \frac{h'(g-1)}{h(g-1)} > \frac{h'(g)}{h(g)}$.

Straightforwardly, the second term, $-\Big(h(g)-h(g-1)\Big)\Big(\frac{h(g-1)}{h(g)}-1\Big) = -\Big(-h(g) - \frac{[h(g-1)]^2}{h(g)}\Big)$, is positive since $h(g) > 0$.

$$R(g) = \frac{1-\rho_0(g)}{1-\rho_1(g)} = \frac{1-\frac{H(g)-1/2}{2-H(g)}}{1-\frac{H(g)-c-1/2}{2-H(g)+c}}$$

is monotonically decreasing in g:

$$\frac{\partial}{\partial g}R(g) \propto \frac{\partial H}{\partial g}\frac{\partial}{\partial H}\log R(g) = \frac{\partial H(g)}{\partial g}\left(\frac{\frac{\partial}{\partial H}(1-\rho_0)}{(1-\rho_0)} - \frac{\frac{\partial}{\partial H}(1-\rho_1)}{(1-\rho_1)}\right) < 0$$
$$\iff \frac{\frac{\partial}{\partial H}(1-\rho_0)}{(1-\rho_0)} - \frac{\frac{\partial}{\partial H}(1-\rho_1)}{(1-\rho_1)} < 0 \text{ because } \frac{\partial H(g)}{\partial g} = h(g) > 0.$$

Because $\frac{\partial H}{\partial g} = h > 0$, we only have to check the sign of the derivative with respect to $H(g)$, treating it as a variable. Observe

$$\frac{\partial}{\partial H}(1-\rho_0) = -\frac{\partial}{\partial H}\frac{H-\frac{1}{2}}{2-H} - \frac{3}{2(2-H)^2}$$
$$\frac{\partial}{\partial H}(1-\rho_1) = -\frac{\partial}{\partial H}\frac{H-\frac{1}{2}-c}{2-H+c} = -\frac{3}{2(2-H+c)^2},$$

so

$$\frac{\frac{\partial}{\partial H}(1-\rho_0)}{(1-\rho_0)} = -\frac{3}{2(2-H)^2}\frac{2-H}{2-H+1/2} = -\frac{3}{2}\frac{1}{(2-H)(2-H+1/2)}$$
$$\frac{\frac{\partial}{\partial H}(1-\rho_1)}{(1-\rho_1)} = -\frac{3}{2(2-H+c)^2}\frac{2-H+c}{2-H+1/2+2c} = -\frac{3}{2}\frac{1}{(2-H+c)(2-H+1/2+2c)}.$$

Notice that $2-H+c>2-H$ and $2-H+1/2+2c>2-H+1/2$, so

$$\frac{1}{(2-H)(2-H+1/2)} > \frac{1}{(2-H+c)(2-H+1/2+2c)}$$
$$\iff \frac{\frac{\partial}{\partial H}(1-\rho_0)}{(1-\rho_0)} < \frac{\frac{\partial}{\partial H}(1-\rho_1)}{(1-\rho_1)}.$$

Thus, $R(g)$ is decreasing in g. ■

Therefore, there exists a unique g^* such that $I(g,g)R(g) \leq \frac{1-q}{q}$ if and only if $g \geq g^*$. Notice that $I(g,g)R(g)$ is decreasing in g and $\frac{1-q}{q}$ is decreasing in q, so $g^*(q)$ is increasing in q.

Let $\rho^*_\omega(q,c) = \rho_\omega(g^*(q,c))$ and $\kappa^*(q,c) = \max\{\hat{\kappa}(g^*(q,c)), 1\}$.

By the same logic as in the previous proof, $\rho^*_\omega(q,c)$ is increasing in q.

To see that $\hat{\kappa}(g^*(q,c))$ is U-shaped in q, notice that

$$\frac{\partial}{\partial g}\hat{\kappa}(g,c) = \frac{\partial}{\partial g}\frac{c}{H(g)-H(g-1)} = -\frac{c[h(g)-h(g-1)]}{[H(g)-H(g-1)]^2} \geq 0 \iff h(g) \leq h(g-1) \iff g \geq 1/2,$$

so $\frac{\partial}{\partial g}\hat{\kappa}(q,c)$ is increasing in g if and only if $g \geq 1/2$. Notice that

$$\frac{\mathrm{d}\hat{\kappa}(g^*(q,c))}{\mathrm{d}q} = \frac{\partial}{\partial g}\hat{\kappa}(g^*(q,c),c)\frac{\partial g^*(q,c)}{\partial g} = -\frac{c[h(g^*(q,c)) - h(g^*(q,c)-1)]}{[H(g^*(q,c)) - H(g^*(q,c)-1)]^2}\underbrace{\frac{\partial g^*(q,c)}{\partial q}}_{>0} \geq 0$$
$$\iff h(g^*(q,c)) - h(g^*(q,c)-1) \leq 0 \iff g^*(q,c) \geq 1/2.$$

Then, $\hat{\kappa}(g^*(q,c))$ is increasing in q if and only if $q > q_{1/2}$ such that $g^*(q_{1/2},c) = 1/2$.

A.3 Proof for Proposition 3

In this proof, we first prove that there exists $g^\dagger$ such that smaller cost of resistance leads to a stricter threshold of the voter if and only if the threshold g^* is high enough ($g^*(c_l) > g^*(c_h) \iff g^*(c_l) > g^\dagger$ when $c_l \in (0, c_h)$). And then, we shows that the same result holds in the limit where c_h is high enough to completely prevent bureaucrats from resisting the reform.

It is sufficient to show that there exists a unique $g^\dagger$ such that $I(g,g)R(g)$ is decreasing in c if and only if $g < g^\dagger$.

Recall

$$\hat{c}(g,c) = \begin{cases} c & \text{if } \hat{\kappa}(g) < 1 \\ H(g) - H(g-1) & \text{if } \hat{\kappa}(g) = 1. \end{cases}$$

With abuse of notation, define

$$\hat{\rho}_1(g,\hat{c}(g,c)) = \begin{cases} \frac{H(g-1)-\hat{c}(g,c)-1/2}{2-H(g-1)+\hat{c}(g,c)} & \text{if } \frac{H(g-1)-\hat{c}(g,c)-1/2}{2-H(g-1)+\hat{c}(g,c)} \geq 0 \\ 0 & \text{if } \frac{H(g-1)-\hat{c}(g,c)-1/2}{2-H(g-1)+\hat{c}(g,c)} < 0. \end{cases}$$

$$\hat{R}(g,\hat{c}(g,c)) = \frac{1-\rho_0(g)}{1-\hat{\rho}_1(g,\hat{c}(g,c))}.$$

For $\hat{\kappa}(g;c) = \frac{c}{H(g)-H(g-1)} < 1 \iff \hat{c}(g,c) = c$,

$$\hat{\rho}_1(g,\hat{c}(g,c)) = \hat{\rho}_1(g,c) = \frac{H(g)-c-1/2}{2-H(g)+c}$$

$$\hat{R}(g,\hat{c}(g,c)) = \hat{R}(g,c) = \frac{1-\hat{\rho}_0(g)}{1-\hat{\rho}_1(g,c)}.$$

If $\hat{\kappa}(g;c) = \frac{c}{H(g)-H(g-1)} \geq 1 \iff \hat{c}(g,c) = H(g)-H(g-1)$, then

$$\hat{\rho}_1(g,\hat{c}(g,c)) = \rho_{1B}(g) = \frac{H(g-1)-1/2}{2-H(g-1)}$$

$$\hat{R}(g,\hat{c}(g,c)) = \frac{1-\hat{\rho}_0(g)}{1-\rho_{B1}(g)}.$$

Also define

$$\hat{I}(g,\hat{c}(g,c)) = \frac{h(g)}{h(g)+\hat{c}(g,c)\frac{h(g-1)-h(g)}{H(g)-H(g-1)}} = \frac{1}{1+\hat{c}(g,c)\frac{[h(g-1)/h(g)]-1}{H(g)-H(g-1)}} = \frac{1}{1+c\varphi(g)}.$$

Again, for $\hat{\kappa}(g;c) = \frac{c}{H(g)-H(g-1)} < 1 \iff \hat{c}(g,c) = c$,

$$\hat{I}(g,\hat{c}(g,c)) = \hat{I}(g,c) = \frac{1}{1+c\frac{[h(g-1)/h(g)]-1}{H(g)-H(g-1)}},$$

and for $\hat{\kappa}(g;c) = \frac{c}{H(g)-H(g-1)} \geq 1 \iff \hat{c}(g,c) = H(g)-H(g-1)$,

$$\hat{I}(g,\hat{c}(g,c)) = I_B(g) = \frac{h(g)}{h(g-1)}.$$

We want to show that there exists a unique $g^\dagger$ such that

$$\hat{I}(g,c_h)\hat{R}(g,c_h) - \hat{I}(g,c_l)\hat{R}(g,c_l) \geq 0 \iff \frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} \geq 0$$

if and only if $g \leq g^\dagger$.

First consider g, c_h, and c_l such that $c_h > c_l > 0$ and $\hat{c}(g,c_h) = c_h$ and $\hat{c}(g,c_l) = c_l$.

Claim 1 *For $c_h < H(g)-H(g-1) \iff \hat{c}(g,c_h) = c_h$, there exists a unique solution $g^\dagger$*

that solves

$$\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} = 0$$

by the Intermediate Value Theorem because

1. $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)}$ *is monotonically decreasing in* g

2. *there exists a small enough* g *such that* $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} > 0$

3. $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} < 0$ *for* $g \geq 1/2$.

Proof for Claim 1.

1. Observe

$$\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} = \frac{1 + c_l\varphi(g)}{1 + c_h\varphi(g)}.$$

is monotonically decreasing in g and takes 1 at $g = 1/2$. To see this, observe

$$\frac{\partial}{\partial g}\frac{1 + c_l\varphi(g)}{1 + c_h\varphi(g)} = \frac{(c_l - c_h)\varphi'(g)}{(1 + c_h\varphi(g))^2}.$$

Recall $\varphi'(g) > 0$ (See the proof for Lemma A2). Because $c_l - c_h < 0$, $\frac{\partial}{\partial g}\frac{1+c_l\varphi(g)}{1+c_h\varphi(g)} < 0$, so $\frac{1+c_l\varphi(g)}{1+c_h\varphi(g)} = \frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)}$ is decreasing in g.

$$\frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} = \frac{1 - \hat{\rho}_1(g;c_h)}{1 - \hat{\rho}_1(g;c_l)} = \frac{1 - \frac{H(g)-c_h-1/2}{2-H(g)+c_h}}{1 - \frac{H(g)-c_l-1/2}{2-H(g)+c_l}} \geq 1$$

is increasing in g. To see that it is increasing in g, observe that

$$\frac{\partial}{\partial H}\frac{1 - \hat{\rho}_1(g;c_h)}{1 - \hat{\rho}_1(g;c_l)} = \frac{[1 - \hat{\rho}_1(g;c_l)]\frac{\partial}{\partial H}[1 - \hat{\rho}_1(g;c_h)] - [1 - \hat{\rho}_1(g;c_h)]\frac{\partial}{\partial H}[1 - \hat{\rho}_1(g;c_l)]}{[1 - \hat{\rho}_1(g;c_l)]^2}.$$

Since $\frac{H(g)-1/2-c_h}{2-H(g)+c_h} < \frac{H(g)-1/2-c_l}{2-H(g)+c_l}$, if $\frac{H(g)-1/2-c_h}{2-H(g)+c_h} > 0$, then $\frac{H(g)-1/2-c_l}{2-H(g)+c_l} > 0$. Suppose

$\frac{H(g)-1/2-c_h}{2-H(g)+c_h} > 0$, so $\hat{\rho}_1(g,c_l) > \hat{\rho}_1(g,c_h) > 0$. Then, $\frac{\partial}{\partial H}\frac{1-\hat{\rho}_1(g;c_h)}{1-\hat{\rho}_1(g;c_l)} \geq 0$

$$\iff [1-\hat{\rho}_1(g;c_l)]\frac{\partial}{\partial H}[1-\hat{\rho}_1(g;c_h)] \geq [1-\hat{\rho}_1(g;c_h)]\frac{\partial}{\partial H}[1-\hat{\rho}_1(g;c_l)]$$
$$\iff [1-\hat{\rho}_1(g;c_l)]\frac{\partial}{\partial H}\hat{\rho}_1(g;c_h) \leq [1-\hat{\rho}_1(g;c_h)]\frac{\partial}{\partial H}\hat{\rho}_1(g;c_l)$$
$$\iff \frac{3[1-\hat{\rho}_1(g;c_l)]}{2(2-H+c_h)^2} \leq \frac{3[1-\hat{\rho}_1(g;c_h)]}{2(2-H+c_l)^2}$$
$$\iff (2-H+c_l)^2-(2-H+c_l)^2\frac{H-c_l-1/2}{2-H+c_l} \leq (2-H+c_h)^2-(2-H+c_h)^2\frac{H-c_h-1/2}{2-H+c_h}$$
$$\iff (2-H+c_l)\Big(2-H+c_l-H+c_l+1/2\Big) \leq (2-H+c_h)\Big(2-H+c_h-H+c_h+1/2\Big)$$
$$\iff (2-H+c_l)\Big(\frac{3}{2}-2H+2c_l\Big) \leq (2-H+c_h)\Big(\frac{3}{2}-2H+2c_h\Big).$$

Notice that $(2-H+c)\Big(\frac{3}{2}-2H+2c\Big)$ is increasing in c for $H\in[0,1]$:

$$\frac{\partial}{\partial c}(2-H+c)\Big(\frac{3}{2}-2H+2c\Big) = \frac{11}{2}-4H+4c = 4(1-H)+\frac{3}{2}+4c > 0$$

Therefore, for $c_h > c_l$, $\frac{\partial}{\partial H}\frac{1-\hat{\rho}_1(g;c_h)}{1-\hat{\rho}_1(g;c_l)} > 0$.

Because $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)}$ is decreasing in g and $\frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)}$ is increasing in g, so $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)}$ is decreasing in g.

Suppose $\frac{H(g)-1/2-c_h}{2-H(g)+c_h} < 0 < \frac{H(g)-1/2-c_l}{2-H(g)+c_l}$, so $\hat{\rho}_1(g,c_l) > \hat{\rho}_1(g,c_h) = 0$. Then, $\frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} = \frac{1}{1-\hat{\rho}_0(g,c_l)}$ is increasing in g since $\hat{\rho}_0(g,c_l)$ is increasing in g. So, $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)}$ is decreasing in g.

Suppose that $\frac{H(g)-1/2-c_l}{2-H(g)+c_l} < 0$, so $\hat{\rho}_1(g,c_l) = \hat{\rho}_1(g,c_h) = 0$. Then, $\frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} = 1$, so $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)}$ is decreasing in g.

Therefore,

$$\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)}$$

is decreasing in g for any g if $c_h < H(g)-H(g-1)$.

2. There exists a small enough g such that $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} > 0$.

   First, $\varphi(g) \geq 0$ if and only if $\frac{h(g-1)}{h(g)} \geq 1$, which holds if and only if $g \geq 1/2$. Thus, for $g < 1/2$, $\varphi(g) < 0$ and $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} = \frac{1+c_l\varphi(g)}{1+c_h\varphi(g)} > 1 \iff 1+c_l\varphi(g) > 1+c_h\varphi(g) \iff c_l < c_h$.

   At the same time, for a small enough g, $\frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} = \frac{1-\hat{\rho}_1(g;c_h)}{1-\hat{\rho}_1(g;c_l)} = 1$ since $\frac{H(g)-1/2-c_l}{2-H(0)+c_l} <$

$0 \iff H(g) - 1/2 - c_l$, so $\hat{\rho}_1(g; c_h) = \hat{\rho}_1(g; c_l) = 0$.[2]

3. Also, there exists a $g > 1/2$ such that $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} - \frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} < 0$. Notice that $\frac{\hat{I}(g,c_h)}{\hat{I}(g,c_l)} \leq 1$ if $g \geq 1/2$ and $\frac{\hat{R}(g,c_l)}{\hat{R}(g,c_h)} = \frac{1-\hat{\rho}_1(g;c_h)}{1-\hat{\rho}_1(g;c_l)} \geq 1$.

■

Consider the case where $\hat{\kappa}(g, c) = 1$, so $\hat{c}(g, c) = H(g) - H(g - 1)$.

Claim 2 *There exists a unique $g^\dagger \leq 1/2$ that solves*

$$\frac{I_B(g)}{\hat{I}(g, c)} - \frac{\hat{R}(g, c)}{R_B(g)} = 0$$

because

1. $\frac{I_B(g)}{\hat{I}(g,c)} \geq 1$ *if and only if* $g \leq 1/2$;
2. $\frac{I_B(g)}{\hat{I}(g,c)}$ *is decreasing in* g *for* $g \leq 1/2$;
3. $\frac{\hat{R}(g,c)}{R_B(g)} \geq 1$. *Also, it is weakly increasing in* g *for* $g \leq 1$.

Proof for Claim 2.

1. Notice that for $\hat{\kappa}(g, c) \in [0, 1]$,

$$\hat{I}(g, c) = \frac{h(g)}{[1 - \hat{\kappa}(g, c)]h(g) + \hat{\kappa}(g, c)h(g - 1)}$$

is a spread of $I_B(g) = \frac{h(g)}{h(g-1)}$ around $g = 1/2$:

$$\begin{aligned} \hat{I}(g, c) \geq I_B(g) &\iff [1 - \hat{\kappa}(g, c)]h(g) + \hat{\kappa}(g, c)h(g - 1) \leq h(g - 1) \\ &\iff h(g) \leq h(g - 1) \iff g \geq 1/2. \end{aligned}$$

2. Observe

$$\frac{I_B(g)}{\hat{I}(g, c)} = \frac{[1 - \hat{\kappa}(g, c)]h(g) + \hat{\kappa}(g, c)h(g - 1)}{h(g - 1)} = \frac{h(g)}{h(g - 1)} + \hat{\kappa}(g, c) - \frac{h(g)}{h(g - 1)}\hat{\kappa}(g, c).$$

Then,

[2] In our model, because the probability of winning the election is 1/2, g such that $\frac{H(0)-1/2-c_l}{2-H(0)+c_l} < 0 \iff 1/2 - [1 - H(g)] - c < 0$ is around $g = 0$. However, for an arbitrary probability of winning the election after the status quo, p, there exists such a g that $p - [1 - H(g)] - c < 0$ by the monotonicity of g.

$$\frac{\partial}{\partial g}\frac{I_B(g)}{\hat{I}(g,c)} = \frac{\partial}{\partial g}\left(\frac{h(g)}{h(g-1)} + \hat{\kappa}(g,c) - \frac{h(g)}{h(g-1)}\hat{\kappa}(g,c)\right)$$
$$= \frac{\partial}{\partial g}\frac{h(g)}{h(g-1)} + \frac{\partial}{\partial g}\hat{\kappa}(g,c) - \frac{\partial}{\partial g}\Big(\frac{h(g)}{h(g-1)}\hat{\kappa}(g,)\Big)$$
$$= \frac{h(g-1)h'(g) - h(g)h'(g-1)}{[h(g-1)]^2} + \frac{c[h(g-1)-h(g)]}{[H(g)-H(g-1)]^2}$$
$$+ \frac{h(g)}{h(g-1)}\frac{c[h(g-1)-h(g)]}{[H(g)-H(g-1)]^2} + \frac{c}{H(g)-H(g-1)}\frac{h(g-1)h'(g)-h(g)h'(g-1)}{[h(g-1)]^2} < 0$$

if and only if

$$\begin{aligned}
&[h(g-1)h'(g) - h(g)h'(g-1)][H(g)-H(g-1)]^2 + c[h(g-1)-h(g)][h(g-1)]^2 \\
&+h(g)c[h(g-1)-h(g)]h(g-1) + c[h(g-1)h'(g)-h(g)h'(g-1)][H(g)-H(g-1)] \\
&= [h(g-1)h'(g)-h(g)h'(g-1)][H(g)-H(g-1)]\Big[c + [H(g)-H(g-1)]\Big] \\
&+c[h(g-1)][h(g-1)-h(g)][h(g-1)+h(g)] < 0.
\end{aligned}$$

Recall that $[h(g-1)h'(g) - h(g)h'(g-1)] < 0$ due to the property of log-concave functions (Bagnoli and Bergstrom, 2006) (See Proof for Proposition 2). Since $[H(g) - H(g-1)]\Big[c + [H(g) - H(g-1)]\Big] > 0$, $[h(g-1)h'(g) - h(g)h'(g-1)][H(g) - H(g-1)]\Big[c + [H(g) - H(g-1)]\Big] < 0$.

Notice that $[h(g-1)-h(g)][h(g-1)+h(g)] = [h(g-1)]^2 - [h(g)]^2 < 0$ for $g < 1/2$ since $[h(g-1)]^2 - [h(g)]^2 < 0 \iff h(g-1) < h(g) \iff g < 1/2$. Since $c[h(g-1)] > 0$, $c[h(g-1)][h(g-1) - h(g)][h(g-1) + h(g)] < 0$ for $g < 1/2$.

3. If $\hat{\kappa}(g,c) \leq 1$, then $\frac{\hat{R}(g,c)}{R_B(g)} \geq 1$. Observe

$$\frac{\hat{R}(g,c)}{R_B(g)} = \frac{\frac{1-\hat{\rho}_0(g)}{1-\hat{\rho}_1(g,c)}}{\frac{1-\hat{\rho}_0(g)}{1-\rho_{B1}(g,c)}} = \frac{1-\rho_{B1}(g,c)}{1-\hat{\rho}_1(g,c)} = \frac{1-\frac{H(g-1)-1/2}{2-H(g-1)}}{1-\frac{H(g)-c-1/2}{2-H(g)+c}} \geq 1$$
$$\iff 1 - \frac{H(g)-c-1/2}{2-H(g)+c} \geq 1 - \frac{H(g-1)-1/2}{2-H(g-1)}$$
$$\iff \frac{H(g-1)-1/2}{2-H(g-1)} \leq \frac{H(g)-c-1/2}{2-H(g)+c}.$$

Since $\frac{T-1/2}{2-T}$ is increasing in T,

$$\frac{H(g-1)-1/2}{2-H(g-1)} \leq \frac{H(g)-c-1/2}{2-H(g)+c} \iff H(g-1) \leq H(g) - c$$
$$\iff c \leq H(g) - H(g-1) \iff \hat{\kappa}(g,c) = \frac{c}{H(g)-H(g-1)} \leq 1.$$

There exists a unique $g_0(c) > 0$ that solves $H(g) - c - 1/2 = 0$. Then $\frac{H(g)-c-1/2}{2-H(g)+c} \leq 0$

for $g \leq g_0(c)$. Notice that $\frac{H(g-1)-1/2}{2-H(g-1)} \leq 0$ for $g \leq 1$. Thus,

$$\hat{\rho}_1(g,c) > \hat{\rho}_{B1}(g) = 0 \text{ if } g \in (g_0(c), 1]$$
$$\hat{\rho}_1(g,c) = \hat{\rho}_{B1}(g) = 0 \text{ if } g \leq g_0(c).$$

Therefore, if $g \in [g_0(c), 1]$, $\frac{\hat{R}(g,c)}{R_B(g)} = \frac{1}{1-\hat{\rho}_1(g,c)} \geq 1$ and is strictly increasing in g. For $g \leq g_c(g)$, $\frac{\hat{R}(g,c)}{R_B(g)} = 1$ and constant.

Argument Solve

$$\frac{I_B(g)}{\hat{I}(g,c)} - \frac{\hat{R}(g,c)}{R_B(g)} = 0.$$

For $g > 1/2$, $\frac{I_B(g)}{\hat{I}(g,c)} < 1$ and $\frac{\hat{R}(g,c)}{R_B(g)} \geq 1$, so $\frac{I_B(g)}{\hat{I}(g,c)} < \frac{\hat{R}(g,c)}{R_B(g)}$.

For $g \leq 1/2$, $\frac{I_B(g)}{\hat{I}(g,c)} \geq 1$ and decreasing in g.

Consider $g_0(c) > 1/2$ first. Since $\hat{\rho}_1(g,c) = 0 \iff \frac{\hat{R}(g,c)}{\hat{R}_B(g)} = \frac{1}{1-\hat{\rho}_1(g,c)} \geq 1$ for $g \in [g_0(c), 1]$ and increasing in g. Since $\frac{I_B(g)}{\hat{I}(g,c)} < 1$ if $g > 1/2$, $\frac{\hat{R}(g,c)}{\hat{R}_B(g)} \geq 1 > \frac{I_B(g)}{\hat{I}(g,c)}$ if $g \geq g_0(c)$. Since $\frac{I_B(g)}{\hat{I}(g,c)}$ is decreasing in g (so gets larger as g decreases) and attains 1 at $g = 1/2$ while $\frac{\hat{R}(g,c)}{\hat{R}_B(g)}$ is fixed at 1 for $g < g_0(c)$, the equation $\frac{I_B(g)}{\hat{I}(g,c)} - \frac{\hat{R}(g,c)}{\hat{R}_B(g)}$ has a unique solution at $g = g^\dagger = 1/2$ by the intermediate value theorem.

Consider $g_0(c) < 1/2$. At $g = g_0(c)$, $\frac{I_B(g)}{\hat{I}(g,c)} > 1 = \frac{\hat{R}(g,c)}{R_B(g)}$. At $g = 1/2$, $\frac{I_B(g)}{\hat{I}(g,c)} = 1 < \frac{\hat{R}(g,c)}{R_B(g)}$. Since $\frac{I_B(g)}{\hat{I}(g,c)}$ is decreasing and $\frac{\hat{R}(g,c)}{R_B(g)}$ is increasing in g for $g \in (g_0(0), 1/2)$, there exists a unique $g^\dagger \in (g_0(0), 1/2)$ such that $\frac{I_B(g)}{\hat{I}(g,c)} \geq \frac{\hat{R}(g,c)}{R_B(g)}$ if and only if $g \leq g^\dagger$ by the intermediate value theorem.

■

A.4 Proof for Proposition 4

Case 1 ($\omega = 0$): It is straightforward that

$$\rho_0^*(q,c) < \rho_{B0}^*(q) \iff \frac{H(g^*(q,c)) - 1/2}{2 - H(g^*(q,c))} < \frac{H(g_B^*(q)) - 1/2}{2 - H(g_B^*(q))}$$

if and only if $g^*(q,c) < g_B^*(q) \iff g^*(q,c) < g^\dagger \iff q < q^\dagger$ since the two ρs are only determined by the comparison between $H(g^*)$ and $H(g_B^*)$ when $\omega = 0$. Namely if $g^* = g_B^*$ ($q = q^\dagger$), the two ρs are the same.
Case 2 ($\omega = 1$): Throughout, focus on the case where bureaucrats resist with an interior

probability:

$$\kappa^*(q^\dagger, c) = \frac{c}{H(g^*) - H(g^* - 1)} < 1 \iff c < H(g^*) - H(g^* - 1).$$

Suppose when $q \geq q^\dagger$, so $g^* \geq g_B^*$. Since $c < H(g^*) - H(g^* - 1) \iff H(g^* - 1) < H(g^*) - c$,

$$\frac{H(g_B^*) - c - \frac{1}{2}}{2 - [H(g_B^*) - c]} = \rho_{B1}^*(q) \leq \frac{H(g^* - 1) - \frac{1}{2}}{2 - H(g^* - 1)} < \frac{H(g^*) - c - \frac{1}{2}}{2 - [H(g^*) - c]} = \rho_1^*(q, c).$$

Notice that even if $q = q^\dagger$, $\rho_{B1}^*(q^\dagger) = \frac{H(g^*-1)-\frac{1}{2}}{2-H(g^*-1)} < \rho_1^*(q, c)$. In other words, $q \leq q^\dagger$ is insufficient to ensure $\rho_{B1}^*(q) \geq \rho_1^*(q)$.

The reason is that bureaucratic resistance affects the incumbent's decision when the reform works ($\omega = 1$) not only through the voter's inference from g (the difference between g^* and g_B^*) but also the direct threat of resistance (the difference between $H(g) - c$ and $H(g - 1)$). In other words, when the reform works, there is an extra restraining force from bureaucrats' resistance through its direct threat.

In turn, it takes a more favorable condition for the reform for resistance leads to a higher incentive for the incumbent to introduce reform in terms of q.

Actually, we can use the fact that $H(g - 1) - 1/2 \leq 0$ for $g \leq 1$, so $\hat{\rho}_{B1}(g) = 0$ for $g \leq 1$.

Recall that there exists a $g_0(c) \in (0, 1]$ that solves $H(g) - c - 1/2 = 0$, so $\hat{\rho}_1(g, c) = 0$ for $g \leq g_0(c)$ as well.

There are two cases depending on whether $g_0(c)$ is larger or smaller than $g^\dagger$. There exists $c^\dagger$ such that $g_0(c)$ that solves $H(g) = c + 1/2$ is strictly less than $g^\dagger$: $g_0(c) < g^\dagger$ if and only if $c < c^\dagger$.

1. Suppose first $c \geq c^\dagger$, so $g_0(c) \geq g^\dagger$. Then,

$$\begin{array}{ll} \begin{array}{l} \rho_0^*(q,c) > \rho_{B0}^*(q) \\ \rho_1^*(q,c) > \rho_{B1}^*(q) = 0 \end{array} & \text{if } g^* > g_0(c) \geq g^\dagger \\ \begin{array}{l} \rho_0^*(q,c) > \rho_{B0}^*(q) \\ \rho_1^*(q,c) = \rho_{B1}^*(q) = 0 \end{array} & \text{if } g^* \in (g^\dagger, g_0(c)) \\ \begin{array}{l} \rho_0^*(q,c) < \rho_{B0}^*(q) \\ \rho_1^*(q,c) = \rho_{B1}^*(q) = 0 \end{array} & \text{if } g^* < g^\dagger. \end{array}$$

2. Suppose now $q < q^\dagger$, so $g^* \leq g^\dagger$.

   We want to show that there exists $q^{\dagger\dagger} < q^\dagger$ such that $\rho_1^*(q, c) = \rho_{B1}^*(q)$ when $q < q^\dagger$ and $\kappa^*(q, c) < 1$ *when c is small enough*, and therefore, $g_0(c)$ that solves $H(g) - c - 1/2 = 0$ is also small enough.

   If $g^* \in (g_0(c), g^\dagger)$, so $g_B^* > g^*$, then $\hat{\rho}_{B1}(g_B^*) = 0 < \hat{\rho}_1(g^*, c)$ while $\hat{\rho}_{B0}(g_B^*) > \hat{\rho}_0(g^*, c)$.

In short,

$$\begin{array}{ll} \rho_0^*(q,c) > \rho_{B0}^*(q) \\ \rho_1^*(q,c) > \rho_{B1}^*(q) = 0 & \text{if } g^* > g^\dagger \\ \rho_0^*(q,c) < \rho_{B0}^*(q) \\ \rho_1^*(q,c) > \rho_{B1}^*(q) = 0 & \text{if } g^* \in (g_0(c), g^\dagger) \\ \rho_0^*(q,c) < \rho_{B0}^*(q) \\ \rho_1^*(q,c) = \rho_{B1}^*(q) = 0 & \text{if } g^* \leq g_0(c). \end{array}$$

Let $q^{\dagger\dagger}$ denote the corresponding q such that $g^*(q,c) = g_0(c)$ in both cases.

A.5 Proof for Proposition 5

Given $\Pr[\omega = 0] = \Pr[\omega = 1] = 1/2$, in the original game, the bureaucrats' ex-ante expected payoff is given by

$$\begin{aligned} & -\frac{(1-\rho_0^*)[1-H(g^*)]}{2}\int_0^1 \kappa d\kappa - \frac{(1-\rho_1^*)}{2}\left(\int_{\kappa^*}^1 \big(\kappa[1-H(g^*)]+c\big)d\kappa - \int_0^{\kappa^*} \kappa[1-H(g^*-1)]d\kappa\right) \\ &= -\frac{(1-\rho_0^*)[1-H(g^*)]}{4} - \frac{(1-\rho_1^*)}{4}\Big([1-H(g^*)](1-\kappa^{*2}) + [1-H(g^*-1)]\kappa^{*2} + c(1-\kappa^*)\Big) \\ &= -\frac{(1-\rho_0^*)[1-H(g^*)]}{4} - \frac{(1-\rho_1^*)}{4}\Big(\kappa^* \cdot \underbrace{\kappa^*\big(H(g^*) - H(g^*-1)\big)}_{=c} + 1 - H(g^*) + c(1-\kappa^*)\Big) \\ &= -\frac{(1-\rho_0^*)[1-H(g^*)]}{4} - \frac{(1-\rho_1^*)}{4}\Big(1 - H(g^*) + c\Big) \end{aligned}$$

when they can resist and

$$\begin{aligned} & -\frac{(1-\rho_{B0}^*)[1-H(g_B^*)]}{2}\int_0^1 \kappa d\kappa - \frac{(1-\rho_{B1}^*)}{2}\int_0^1 \kappa[1-H(g_B^*-1)]d\kappa \\ &= -\frac{(1-\rho_{B0}^*)[1-H(g_B^*)] + (1-\rho_{B1}^*)[1-H(g_B^*-1)]}{4} \end{aligned}$$

when they cannot. Thus, they are better off being able to resist if and only if

$$DU_B := \left(\begin{array}{l} (1-\rho_{B0}^*)[1-H(g_B^*)] - (1-\rho_0^*)[1-H(g^*)] + \\ (1-\rho_{B1}^*)[1-H(g_B^*-1)] - (1-\rho_1^*)[1-H(g^*)+c] \end{array} \right) > 0.$$

First, notice $(1-\rho_{B\omega}^*)$ and $(1-\rho_\omega^*)$ are *decreasing* in g_B^* and g^* respectively as well as $1-H(g_B^*-\omega)$ and $1-H(g_B^*-\omega)$, and $1-H(g^*-\omega)$ and $1-H(g^*-\omega)$.

Suppose that $q \leq q^{\dagger\dagger} < q^\dagger$. Then, the resistance makes the voter more lenient and leads to over-reform: $g_B^* > g^*$, $\rho_{B1}^* = \rho_1^* = 1$, and $\rho_{B0}^* \geq \rho_0^*$ (Recall Proposition 3 and Proposition 4). Then,

$$DU_B = (1-\rho_{B0}^*)[1-H(g_B^*)] - (1-\rho_0^*)[1-H(g^*)].$$

Since $\rho^*_{B0} \geq \rho^*_0$, $(1-\rho^*_{B0}) \leq (1-\rho^*_0)$. Also, because $g^*_B > g^*$, $[1-H(g^*_B)] < [1-H(g^*)]$. As a result, $DU_B < 0$ when $q \leq q^{\dagger\dagger}$: bureaucrats are better off being able to tie their hands when the reform is popular.

Suppose that $q \geq q^\dagger$, so $g^*_B > g^*$, $\rho^*_{B1} < \rho^*_1$, and $\rho^*_{B0} < \rho^*_0$. Then, in contrast, $(1-\rho^*_{B0})[1-H(g^*_B)] > (1-\rho^*_0)[1-H(g^*)]$ and $(1-\rho^*_{B1})[1-H(g^*_B-1)] > (1-\rho^*_1)[1-H(g^*)+c]$, so $DU_B > 0$: bureaucrats are better off being able to resist if the reform is relatively unpopular.

Given that $\Pr[\omega=1]=1/2$, the incumbent's expected payoff in the original game is

$$\frac{1}{2}\left(\int_0^{\rho^*_0}\frac{1}{2}d\rho + \int_{\rho^*_0}^1 \rho+(1+\rho)[1-H(g^*)]d\rho + \int_0^{\rho^*_1}\frac{1}{2}d\rho + \int_{\rho^*_1}^1 \rho+(1+\rho)[1-H(g^*)+c]d\rho\right)$$

when bureaucrats can resist (Recall that, in equilibrium, the incumbent's reelection probability in $\omega=1$ is $1-H(g^*)+c=(1-\kappa^*)[1-H(g^*)]+\kappa^*[1-H(g^*-1)]=1-H(g^*)+\kappa^*[H(g^*)-H(g^*-1)]$ since $\kappa^*=\frac{c}{H(g^*)-H(g^*-1)}$.) and

$$\frac{1}{2}\left(\int_0^{\rho^*_{B0}}\frac{1}{2}d\rho + \int_{\rho^*_{B0}}^1 \rho+(1+\rho)[1-H(g^*_B)]d\rho + \int_0^{\rho^*_{B1}}\frac{1}{2}d\rho + \int_{\rho^*_{B1}}^1 \rho+(1+\rho)[1-H(g^*_B-1)]d\rho\right)$$

when bureaucrats can resist.

Suppose $q \leq q^{\dagger\dagger}$, so $\rho^*_1=\rho^*_{B1}=0$, $g^* < g^*_B$ and $\rho^*_0 \leq \rho^*_{B0}$. Then, the incumbent's net gain from the bureaucrats' ability to resist is positive. To see this, first look into the part where $\omega=0$.

$$\begin{aligned}
&\int_0^{\rho^*_0}\frac{1}{2}d\rho + \int_{\rho^*_0}^1 \rho+(1+\rho)[1-H(g^*)]d\rho - \left(\int_0^{\rho^*_{B0}}\frac{1}{2}d\rho + \int_{\rho^*_{B0}}^1 \rho+(1+\rho)[1-H(g^*_B)]d\rho\right)\\
&= \int_{\rho^*_0}^{\rho^*_{B0}}\left(\rho+(1+\rho)[1-H(g^*)]-\frac{1}{2}\right)d\rho + \int_{\rho^*_{B0}}^1 (1+\rho)\underbrace{[H(g^*_B)-H(g^*)]}_{>0 \iff g^*_B > g^*.}d\rho
\end{aligned}$$

Recall that in equilibrium, $\rho+(1+\rho)[1-H(g^*)] > \frac{1}{2} \iff \rho > \rho^*_0$, so the net gain is positive if $\omega=0$. Now, consider the part for $\omega=1$.

$$\begin{aligned}
&\int_0^{\rho^*_1}\frac{1}{2}d\rho + \int_{\rho^*_1}^1 \rho+(1+\rho)[1-H(g^*)+c]d\rho - \int_0^{\rho^*_{B1}}\frac{1}{2}d\rho + \int_{\rho^*_{B1}}^1 \rho+(1+\rho)[1-H(g^*_B-1)]d\rho\\
&= \int_{\rho^*_1}^{\rho^*_{B1}}\left(\rho+(1+\rho)[1-H(g^*)]+c-\frac{1}{2}\right)d\rho + \int_{\rho^*_{B1}}^1 (1+\rho)[H(g^*_B-1)-H(g^*)+c]d\rho\\
&= \int_0^1 (1+\rho)[H(g^*_B-1)-H(g^*)+c]d\rho > 0
\end{aligned}$$

since $[H(g^*_B-1)-H(g^*)+c] > 0$ because $H(g^*_B-1)-H(g^*-1) > 0$ and $c=\kappa^*[1-H(g^*-1)]+(1-\kappa^*)[1-H(g^*)]=[1-H(g^*)]+\kappa^*[H(g^*)-H(g^*-1)] > H(g^*)-H(g^*-1)$, so $c-H(g^*) > H(g^*-1)$.

In contrast, suppose $q \geq q^\dagger$, so $\rho_1^* > \rho_{B1}^*$, $g^* > g_B^*$ and $\rho_0^* > \rho_{B0}^*$. In $\omega = 0$,

$$\int_0^{\rho_0^*} \frac{1}{2} d\rho + \int_{\rho_0^*}^1 \rho + (1+\rho)[1 - H(g^*)] d\rho - \left(\int_0^{\rho_{B0}^*} \frac{1}{2} d\rho + \int_{\rho_{B0}^*}^1 \rho + (1+\rho)[1 - H(g_B^*)] d\rho \right)$$
$$= \int_{\rho_{B0}^*}^{\rho_0^*} \left(\rho + (1+\rho)[1 - H(g^*)] - \frac{1}{2} \right) d\rho + \int_{\rho_0^*}^1 (1+\rho) \underbrace{[H(g_B^*) - H(g^*)]}_{<0 \iff g_B^* < g^*.} d\rho < 0$$

since $\rho + (1+\rho)[1 - H(g^*)] < \frac{1}{2} \iff \rho < \rho_0^*$. In $\omega = 1$,

$$\int_0^{\rho_1^*} \frac{1}{2} d\rho + \int_{\rho_1^*}^1 \rho + (1+\rho)[1 - H(g^*) + c] d\rho - \int_0^{\rho_{B1}^*} \frac{1}{2} d\rho + \int_{\rho_{B1}^*}^1 \rho + (1+\rho)[1 - H(g_B^* - 1)] d\rho$$
$$= \int_{\rho_{B1}^*}^{\rho_1^*} \left(\rho + (1+\rho)[1 - H(g^*) + c] - \frac{1}{2} \right) d\rho + \int_{\rho_1^*}^1 (1+\rho)[H(g_B^* - 1) - H(g^*) + c] d\rho < 0$$

since $\rho + (1+\rho)[1 - H(g^*) + c] - \frac{1}{2} < 0$ iff $\rho < \rho_1^*$ in equilibrium, and $H(g_B^* - 1) - H(g^*) + c < 0 \iff c = H(g^*) - H(g^* - 1) < H(g^*) - H(g_B^* - 1)$ given that $H(g^* - 1) > H(g_B^* - 1)$ because $g^* > g_B^*$.

A.6 Proof for Proposition 6

Suppose that the voter reelects the incumbent if and only if $g \geq g'$ every period. By assumption 2, bureaucrats resist with with probability $1 - \hat{\kappa}(g')$ where

$$\hat{\kappa}(g') = \begin{cases} \frac{c}{H(g') - H(g'-1)} & \text{if } H(g') - H(g'-1) \geq c \\ 1 & \text{if } H(g') - H(g'-1) < c. \end{cases}$$

Then, for $\hat{\kappa}(g') \leq 1$, the pro-reform incumbent gets $[1 - H(g') + \omega c]\rho + \rho$ in every period if reform is in place and 1/2 if not just like in the main model. So, she uses the same decision rule as in the main model because every term is simply multiplied by $\frac{1}{1-\delta}$. Again, as in the original model,

$$\hat{\rho}(g'; \omega) = \begin{cases} \frac{H(g') - 1/2}{2 - H(g')} & \text{if } \omega = 0 \\ \frac{H(g') - 1/2 - c}{2 - H(g') + c} & \text{if } \omega = 1 \end{cases}$$

she chooses the reform if and only if $\rho \geq \hat{\rho}(g'; \omega)$ in ω.

Then, when reform is in place, the voter forms a posterior belief from g_t

$$\mu_t(g_t; g') = \frac{\mu_{t-1}[1 - \hat{\rho}(g'; 1)] \big(\hat{\kappa}(g') h(g_t - 1) + [1 - \hat{\kappa}(g')] h(g_t) \big)}{\mu_{t-1}[1 - \hat{\rho}(g'; 1)] \big(\hat{\kappa}(g') h(g_t - 1) + [1 - \hat{\kappa}(g')] h(g_t) \big) + (1 - \mu_{t-1})[1 - \hat{\rho}(g'; 0)] h(g_t)}$$

and reelects the pro-reform incumbent if and only if $g_t \geq g_E^*$ that solves

$$\hat{\kappa}(g_t) \cdot \mu_t(g_t) \geq q \iff \mu_t(g_t) \geq \frac{q}{\hat{\kappa}(g_t)} \tag{A4}$$

with equality in every t. In words, the voter applies the threshold g_E^* in the election at period t with the expectation that the probability of bureaucrats' resistance in $t+1$ is just like as in t. Also, for the voter to apply the same threshold in t as in $t-1$, $\mu_t(g_E^*) = \mu_{t-1}$. Both condition holds by g_E^* that solves

$$\frac{1-\hat{\rho}(g;0)}{1-\hat{\rho}(g;1)} \frac{h(g)}{\hat{\kappa}(g)h(g-1)+[1-\hat{\kappa}(g)]h(g)} = 1. \tag{A5}$$

To see this, observe first

$$\mu_t(g_E^*) = \frac{1}{1+\frac{1-\mu_{t-1}}{\mu_{t-1}}\frac{1-\hat{\rho}(g_E^*;0)}{1-\hat{\rho}(g_E^*;1)}\frac{h(g_E^*)}{\hat{\kappa}(g_E^*)h(g_E^*-1)+[1-\hat{\kappa}(g_E^*)]h(g_E^*)}} = \mu_{t-1}$$

holds with g_E^* that solves (A5). Second, by the logic of steady state, $t \geq 2$, $\mu_t(g_E^*) = \mu_{t-1} = \frac{q}{\hat{\kappa}(g_E^*)}$, so $\frac{1-\mu_{t-1}}{\mu_{t-1}} = \frac{\frac{\hat{\kappa}(g_E^*)-q}{\hat{\kappa}(g_E^*)}}{\frac{q}{\hat{\kappa}(g_E^*)}} = \frac{\hat{\kappa}(g_E^*)-q}{q}$. If (A5) holds, then

$$\mu_t(g_E^*) = \frac{1}{1+\frac{\hat{\kappa}(g_E^*)-q}{q}\frac{1-\hat{\rho}(g_E^*;0)}{1-\hat{\rho}(g_E^*;1)}\frac{h(g_E^*)}{\hat{\kappa}(g_E^*)h(g_E^*-1)+[1-\hat{\kappa}(g_E^*)]h(g_E^*)}} = \frac{1}{1+\frac{\hat{\kappa}(g_E^*)-q}{q}} = \frac{1}{\frac{\hat{\kappa}(g_E^*)}{q}} = \frac{q}{\hat{\kappa}(g_E^*)}.$$

Now let's compare g_E^* to g_{EB}^*, the voter's stated-state threshold in the game without bureaucratic resistance that solves $\mu_t(g_t) \geq q$ with equality. Again, the logic of steady state enforces $\mu_t(g_{EB}^*) = \mu_{t-1}$ to hold for every $t \geq 1$. Recall that without resistance,

$$\mu_t(g) = \frac{1}{1+\frac{1-\mu_{t-1}}{\mu_{t-1}}\frac{1-\hat{\rho}(g;0)}{1-\hat{\rho}(g;1)}\frac{h(g)}{h(g-1)}} = \mu_{t-1},$$

so g_{EB}^* solves

$$\frac{1-\mu_{t-1}}{\mu_{t-1}}\frac{1-\hat{\rho}(g;0)}{1-\hat{\rho}(g;1)}\frac{h(g)}{h(g-1)} = 1 \tag{A6}$$

Notice that g^* that solves (A5) and g_{EB}^* that solves (A6) are the solutions in the original game when $q = 1/2$. Recall that $q^\dagger \leq 1/2$. Thus, in the steady state where the voter's belief is fixed as $\mathbb{E}[\omega|g^*]$ with g^* that solves (A5) or (A6) depending on whether the bureaucrats can resist or not, the voter is stricter when bureaucrats can resist than when they cannot, $g_E^* > g_{EB}^*$.

B Dynamics of Policy-Making

First, we show that in our two-period setup, the incumbent prefers to have a commitment power to stick with the status quo, and the voter also supports it. We then discuss the logic that supports the policy continuity in the dynamic game extended beyond the two periods.

B.1 If the Incumbent Can Commit, She Will (and the Voter Will Support That) in a Two-Period Model

Assume that the incumbent *cannot* commit to the status quo in $t = 2$. The voter then knows that the incumbent will choose the reform in $t = 2$ regardless of her policy in $t = 1$. Therefore, he only reelects the incumbent if he believes, based on his observation of g, that the reform is better than the status quo (i.e., $\mathrm{E}[\omega|g] \geq q$). Since the voter is more likely to observe a high g when $\omega = 1$ and the reform is implemented, the incumbent's electoral incentives to choose the reform is weakly larger when it is effective than when it is not.[3] As a result, the incumbent's decision signals ω: $\omega = 1$ is more likely when she chooses the reform than the status quo, and the incumbent cannot choose the status quo in $t = 1$ without damaging the voter's expectation about the reform's worth, as well has her reelection prospects.[4] Consequently, without commitment, we have an *unraveling* result (Milgrom, 1981) where incumbents *always* choose the reform, making the analysis trivial.

From this perspective, the commitment to the status quo benefits the incumbent and the voter. The incumbent can cut her electoral loss when $\omega = 0$. In turn, the expected value of the *introduced* reform increases because when $\omega = 0$, the incumbent can choose the status quo with the commitment. This benefits both the voter and the reforming incumbent. Therefore, we can expect the incumbent to develop a commitment device to tie her hands once she has chosen the status quo, and the voter supports it.

B.2 The Logic Behind Policy Continuity in a Dynamic Game with More than Two Periods

We now explain why, in our setting, the incumbent is unlikely to reverse her policy choice even when the game is extended beyond the second period, $t \geq 2$.

Assume that the following holds:

- a pro-reform politician is always weakly more likely to implement the reform that works ($\omega = 1$) than the one that does not ($\omega = 0$).
- the voter conditions his election decision at t based on the reform's expected value in every period, $\mu_t \in [0, 1]$; he strictly prefers the pro-reform politician in the election in period t iff μ_t is high enough.

In our model, the first property holds because g is informative about the reform's value. More broadly, this property holds whenever the incumbent has stronger incentives to implement effective reforms than ineffective ones—whether due to informational advantages (as in our model) or intrinsic preferences for good policy. The key is that choosing not to implement the reform adversely affects the voter's belief about ω. The second property holds if the voter believes that the pro-reform incumbent is more likely to implement or continue the reform than the anti-reform challenger. Under these conditions, there exists an equilibrium where the incumbent keeps the policy once she selects it, and the voter expects this behavior.

[3] Namely, there is no such equilibrium where the incumbent is *less* likely to choose the reform when it is effective than when it is not.

[4] Formally, $\mathrm{E}[\omega|a = 1] \geq \mathrm{E}[\omega|a = 0]$ if $\Pr[a = 1|\omega = 1] \geq \Pr[a = 1|\omega = 0]$ for any interior $\Pr[\omega = 1]$.

To see why the incumbent cannot save the reform for later periods, suppose the incumbent chooses the status quo in period t. This choice worsens the appeal of reform in $t+1$ because it reveals negative information about the reform's value. Namely, when the pro-reform incumbent does not implement the reform in t, the game in $t+1$ starts with a lower belief, as long as the incumbent is more likely to implement effective reforms than ineffective ones. Formally,

$$\mu_{t+1} = \mathbb{E}[\omega|\text{no reform}_t] = \frac{\mu_t \Pr[\text{no reform}_t|\omega = 1]}{\mu_t \Pr[\text{no reform}_t|\omega = 1] + (1-\mu_t)\Pr[\text{no reform}_t|\omega = 0]} \leq \mu_t$$

iff $\Pr[\text{no reform}_t|\omega = 1] \leq \Pr[\text{no reform}_t|\omega = 1]$. This reduction in belief increases the performance threshold the voter requires to reelect a reforming incumbent in the next election, thereby lowering the ex-ante probability of reelection for a reforming incumbent in $t+1$. Suppose, for simplicity, that bureaucrats do not resist. Then, the voter's posterior belief in $t+1$ given g_{t+1}, conditional on the reform being implemented in $t+1$, is increasing in μ_t:

$$\begin{aligned}\mu_{t+1}(g_{t+1};\text{reform}_{t+1}) &= \frac{\mu_t \Pr[\text{reform}_{t+1}|\omega = 1]h(g_{t+1}-1)}{\mu_t \Pr[\text{reform}_{t+1}|\omega = 1]h(g_{t+1}-1) + (1-\mu_t)\Pr[\text{reform}_{t+1}|\omega = 0]h(g_{t+1})} \\ &= \frac{1}{1+\frac{1-\mu_t}{\mu_t}\frac{\Pr[\text{reform}_{t+1}|\omega=0]}{\Pr[\text{reform}_{t+1}|\omega=1]}\frac{h(g_{t+1})}{h(g_{t+1}-1)}}.\end{aligned}$$

Recall that in the main game, the voter reelects the reforming incumbent in the election if and only if

$$\begin{aligned}&\frac{1-\mu_t}{\mu_t}\frac{\Pr[\text{reform}_{t+1}|\omega = 0]}{\Pr[\text{reform}_{t+1}|\omega = 1]}\frac{h(g_{t+1})}{h(g_{t+1}-1)} \leq \frac{1-q}{q} \\ \iff &\frac{\Pr[\text{reform}_{t+1}|\omega = 0]}{\Pr[\text{reform}_{t+1}|\omega = 1]}\frac{h(g_{t+1})}{h(g_{t+1}-1)} \leq \frac{1-q}{q}\frac{\mu_t}{1-\mu_t}.\end{aligned}$$

So, decreasing μ_t has the same effect as increasing q for the incumbent's reelection, i.e., both make reelection harder.[5] In turn, the incumbent is *less* likely to get reelected after implementing the reform in period $t+1$ than in period t if the pro-reform incumbent in t did not implement the reform since $\mu_{t+1} \leq \mu_t$. This implies that if the reform is not attractive to the pro-reform incumbent and she chooses the status quo in t, the reform will be even less attractive in $t+1$.

The converse also holds: when the pro-reform incumbent wins reelection in t after implementing reform, reelection incentives in $t+1$ will encourage reform continuation. Winning in t requires voter beliefs high enough to make continuing reform attractive in $t+1$.

Consider a three-period game where $\mu_0 = q$. The voter reelects the reforming incumbent in $t = 1$ iff $\mu_1 \geq q$.[6] If the incumbent wins after implementing reform, then $\mu_1 \geq \mu_0 =$

[5]Here, we assume that the voter's decision rule is the same as in the original model. However, this is not necessary; in general, the voter will reelect the incumbent with any g_t that would lead to reelection under a lower belief μ_t' when holding a higher belief μ_t'', but not vice versa. Formally, when the voter reelects the incumbent in t iff $g_t \in G(\mu_t)$ where $G(\mu_t) \subseteq \mathbb{R}$ is the *reelection set*, the size of $G(\mu_t)$ is increasing in μ_t: For $\mu_t' < \mu_t''$, we have $G(\mu_t') \subseteq G(\mu_t'')$.

[6]The inequality holds even when the incumbent continues reform in $t+1$ with probability $z \in (0,1]$ while

q, meaning she faces a more favorable reelection environment in the second cycle. Thus, an incumbent who reforms and wins reelection will continue reform in the second term. Conversely, beliefs that would discourage second-period reform ($\mu_1 < \mu_0 = q$) arise only when reform causes the incumbent to lose the first election.

Therefore, a reforming incumbent will not revert to the status quo at $t = 1$ if she wins reelection, because electoral victory requires $\mu_1 \geq q$—which makes reform at least as attractive as in $t = 0$. While reform at $t = 0$ could produce beliefs low enough to discourage continuation at $t = 1$, the incumbent would lose the election and lack the opportunity to change policy. An incumbent in office at $t = 1$ after reform necessarily has $\mu_1 \geq q$; being in office with $\mu_1 < q$ is off-the-equilibrium-path.

$\mu_0 = q$

$\mu_1(\text{reform}_{t=1}, \text{high } g_1)$:
The reforming incumbent *wins* the election.
The incumbent *wants* to implement the reform for the next election.

$\mu_1(\text{reform}_{t=1}, \text{low } g_1)$:
The reforming incumbent *loses* the election.
The incumbent *does not want* to implement the reform for the next election.
But she cannot because she lost the election.

Taken together, unless driven by exogenous factors or a set of assumptions under which the two conditions at the beginning of this subsection do not hold, incumbent politicians who win reelection after implementing a successful reform would not revert to the previous status quo voluntarily. The reform's success in the previous term makes voters more favorable toward continuing it, creating strong incentives for the incumbent to maintain the policy. Conversely, when politicians attempt to revive old initiatives that previously failed or performed poorly, voters are typically less receptive than they were when those initiatives were first introduced. Therefore, politicians either push forward with reforms and stick to them, or they abandon reforms and maintain the status quo. Politicians rarely bounce between the two policies, as each choice creates self-reinforcing electoral incentives that lock them into a particular policy trajectory via electoral incentives.

The incumbent's incentive to maintain policy consistency aligns with existing literature. First, using a two-period accountability model, Andreottola (2021) shows how electoral accountability discourages incumbent politicians from shifting policy across terms. In his model, the voter cares about the incumbent's competence—specifically, the quality of her private information about the state of the world. When states of the world across the two periods are positively correlated, flip-flopping signals that the incumbent's first policy decision may have been incorrect, revealing potential incompetence. This result also coheres with Moskowitz, Rogowski and Snyder (2024), who find that changes in policy positions in the US Congress are driven primarily by compositional changes (member turnover) rather than behavioral changes among existing members. Moreover, the assumption that politicians rarely reverse their policy decisions is also common in seminal models of politician ideal points. For instance, Poole and Rosenthal (2000) adopt the continuity assumption that

the challenger reverts with certainty, since the myopic voter supports the incumbent iff $z\mu_t + (1-z)q \geq q \iff (1-z)\mu_t \geq (1-z)q \iff \mu_t \geq q$.

"politicians would rather go down with the ship than switch to another ship" to measure policy positions over time using DW-NOMINATE.

C Robustness: If Resistance Damages Ineffective Reform

Suppose that the bureaucrats can resist an ineffective reform and horizontally shift the density of g from $h(g)$ to $h(g-\alpha)$ such that $\alpha \geq 0$.

Define

$$\kappa_0(g) = \begin{cases} \frac{c}{H(g+\alpha)-H(g)} & \text{if } \frac{c}{H(g+\alpha)-H(g)} \leq 1 \\ 1 & \text{if } \frac{c}{H(g+\alpha)-H(g)} > 1 \end{cases}$$

$$\kappa_1(g) = \begin{cases} \frac{c}{H(g)-H(g-1)} & \text{if } \frac{c}{H(g)-H(g-1)} \leq 1 \\ 1 & \text{if } \frac{c}{H(g)-H(g-1)} > 1 \end{cases}$$

$$I_\alpha(g,c) = \frac{\kappa_0 h(g) + (1-\kappa_0)h(g+\alpha)}{\kappa_1 h(g-1) + (1-\kappa_1)h(g)}$$

$$\rho_{\alpha 0}(g,c) = \begin{cases} \frac{H(g+\alpha)+\frac{1}{2}-\kappa_0[H(g+\alpha)+H(g)]}{2-H(g+\alpha)+\kappa_0[H(g+\alpha)+H(g)]} & \text{if } \frac{H(g+\alpha)+\frac{1}{2}-\kappa_0[H(g+\alpha)+H(g)]}{2-H(g+\alpha)+\kappa_0[H(g+\alpha)+H(g)]} \in [0,1] \\ 1 & \text{if } \frac{H(g+\alpha)+\frac{1}{2}-\kappa_0[H(g+\alpha)+H(g)]}{2-H(g+\alpha)+\kappa_0[H(g+\alpha)+H(g)]} > 1 \\ 0 & \text{if } \frac{H(g+\alpha)+\frac{1}{2}-\kappa_0[H(g+\alpha)+H(g)]}{2-H(g+\alpha)+\kappa_0[H(g+\alpha)+H(g)]} < 0 \end{cases}$$

$$\rho_{\alpha 1}(g,c) = \begin{cases} \frac{H(g)+\frac{1}{2}-\kappa_1[H(g)-H(g-1)]}{2-H(g)+\kappa_1[H(g)-H(g-1)]} & \text{if } \frac{H(g)+\frac{1}{2}-\kappa_1[H(g)-H(g-1)]}{2-H(g)+\kappa_1[H(g)-H(g-1)]} \in [0,1] \\ 1 & \text{if } \frac{H(g)+\frac{1}{2}-\kappa_1[H(g)-H(g-1)]}{2-H(g)+\kappa_1[H(g)-H(g-1)]} > 1 \\ 0 & \text{if } \frac{H(g)+\frac{1}{2}-\kappa_1[H(g)-H(g-1)]}{2-H(g)+\kappa_1[H(g)-H(g-1)]} < 0 \end{cases}$$

$$R_\alpha(g,c) = \frac{1-\rho_{\alpha 0}(g,c)}{1-\rho_{\alpha 1}(g,c)}.$$

Lemma A3 *For g such that $\kappa_0(g) \geq \kappa_1(g)$, $I_\alpha(g,c)$ and $R_\alpha(g,c)$ are decreasing in g.*

Proof. For x, y such that $x \leq y$, define

$$\hat{\kappa}(g;x,y) := \begin{cases} \frac{c}{H(g+x)-H(g-y)} & \text{if } \frac{c}{H(g+x)-H(g-y)} \leq 1 \\ 1 & \text{if } \frac{c}{H(g+x)-H(g-y)} > 1 \end{cases}$$

$$\iota(g;x,y) := h(g+x) + \hat{\kappa}(g;x,y)[h(g-y) - h(g+x)]$$

$$\hat{\rho}(g;x,y) := \frac{H(g+x) + \frac{1}{2} - \hat{\kappa}(g;x,y)[H(g+x) - H(g-y)]}{2 - H(g+x) + \hat{\kappa}(g;x,y)[H(g+x) - H(g-y)]}.$$

$$I_\alpha(g) = \frac{\iota(g;\alpha,0)}{\iota(g;0,1)} \qquad R_\alpha(g) = \frac{1-\hat{\rho}(g;\alpha,0)}{1-\hat{\rho}(g;0,1)}$$

$$I(g) = \frac{\iota(g;0,0)}{\iota(g;0,1)} \qquad R(g) = \frac{1-\hat{\rho}(g;0,0)}{1-\hat{\rho}(g;0,1)}$$

$I(g)R(g) = \frac{1-q}{q}$ is equivalent to

$$\log \iota(g;0,0) - \log \iota(g;0,1) + \log[1-\hat{\rho}(g;0,0)] - \log[1-\hat{\rho}(g;0,1)] = \log \frac{1-q}{q}$$

and $I_\alpha(g)R_\alpha(g) = \frac{1-q}{q}$ is equivalent to

$$\log \iota(g;\alpha,0) - \log \iota(g;0,1) + \log[1-\hat{\rho}(g;\alpha,0)] - \log[1-\hat{\rho}(g;0,1)] = \log \frac{1-q}{q}.$$

Notice that $\log \iota(g;\alpha,0)$ is a horizontal shift of $\log \iota(g;0,0)$ to the right and $\log[1-\hat{\rho}(g;\alpha,0)]$ is a horizontal shift of $\log[1-\hat{\rho}(g;0,0)]$ to the right for g such that $\hat{\kappa}(g;\alpha,0) \geq (\hat{\kappa}(g;0,1)$, so $I_\alpha(g)R_\alpha(g)$ is a horizontal shift of $I(g)R(g)$ to the right. Because a horizontal shift of a monotone function is monotone, $I_\alpha(g)R_\alpha(g)$ is monotone for g such that $\hat{\kappa}(g;\alpha,0) \geq \hat{\kappa}(g;0,1)$. ■

Lemma A4 *For g such that $\kappa_0(g) \geq \kappa_1(g)$, there exists a unique $g_\alpha^\dagger$ $I_\alpha(g)R_\alpha(g) \geq I_B(g)R_B(g)$.*

Proof. $I_\alpha(g)R_\alpha(g) = I_B(g)R_B(g)$ if and only if

$$\frac{I_\alpha(g)}{I_B(g)} = \frac{R_B(g)}{R_\alpha(g)} \iff \frac{\iota(g;\alpha,0)}{\iota(g;0,1)}\frac{\iota(g;1,1)}{\iota(g;0,0)} = \frac{1-\hat{\rho}(g;0,0)}{1-\hat{\rho}(g;1,1)}\frac{1-\hat{\rho}(g;0,1)}{1-\hat{\rho}(g;\alpha,0)}.$$

By the same logic as in the proof above, if $\hat{\kappa}(g;\alpha,0) \geq \hat{\kappa}(g;0,1)$, $\frac{I_\alpha(g)}{I_B(g)}$ and $\frac{R_B(g)}{R_\alpha(g)}$ are respectively horizontal shifts of $\frac{I(g)}{I_B(g)}$ and $\frac{R_B(g)}{R(g)}$, which are monotone in g. ■

Lemma A5 *For g such that $\kappa_1 \leq 1$, there here exists a $\alpha^\dagger \in (0,1)$ such that $\kappa_1 \leq \kappa_0$ if $\alpha \in [0, \alpha^\dagger]$.*

Proof. $\kappa_1 \leq \kappa_0$ iff $H(g) - H(g-1) \geq H(g+\alpha) - H(g) \iff 2H(g) \geq H(g+\alpha) + H(g-1)$. Notice that $H(g+\alpha)+H(g-1)$ is monotonically increasing in g and α and less than $2H(g)$ for any g when $\alpha = 0$ (by the FOSD), so there exists $\alpha^\dagger$ such that there exists g such that $2H(g) < H(g+\alpha) + H(g-1)$ if $\alpha > \alpha^\dagger$. ■

Proposition A1 *For $\alpha \in [0, \alpha^\dagger)$,*

1. *$\kappa_1(g) \leq \kappa_0(g)$;*
2. *there exists a unique pure strategy equilibrium exists analogous to the main model;*
3. *there exists unique $q_\alpha^\dagger$ such that $g_\alpha^* > g_B^*$.*

Proof. Results follow from lemmas above. ■

D Countervailing Effects of Resistance on Voter Learning

D.1 Illustration of Learning Effects

As the incumbent can strategically choose whether to introduce reform or not and bureaucrats can resist reform, g is an *obfuscated* signal of the reform's true value of ω. To understand the effect of strategic obfuscation on the voter's learning, consider the benchmark case where neither player intervenes with g, and the voter observes $g = x + \eta$.

Suppose that, for an arbitrary cutoff g', the voter concludes that the reform will work if he observes a "positive" signal $g \geq g'$ and it will not work if he observes a "negative" signal $g < g'$. Then, we can define four events, shown in Table 1.

Table 1: Confusion Matrix for Voter Inference

		Prediction	
		$g < g'$	$g > g'$
Actual condition	$\omega = 1$	FN	TP
	$\omega = 0$	TN	FP
		False omission rate (FOR) $\frac{FN}{TN+FN}$	Positive predictive value (PPV) $\frac{TP}{TP+FP}$

Notes: FN denotes false negatives; TN denotes true negatives; TP denotes true positives; FP denotes false positives.

The voter faces a Goldilocks problem in choosing the optimal g', i.e., he cannot be either too lenient or too stringent. If he is too lenient and chooses a low g', then a positive signal $g \geq g'$ does not necessarily mean that the reform outperforms the status quo. Thus, he wants to pick a high enough g' so that the positive predictive value (PPV), i.e.

$$\Pr[\omega = 1 | g \geq g'] = \frac{\Pr[TP]}{\Pr[TP] + \Pr[FP]}$$

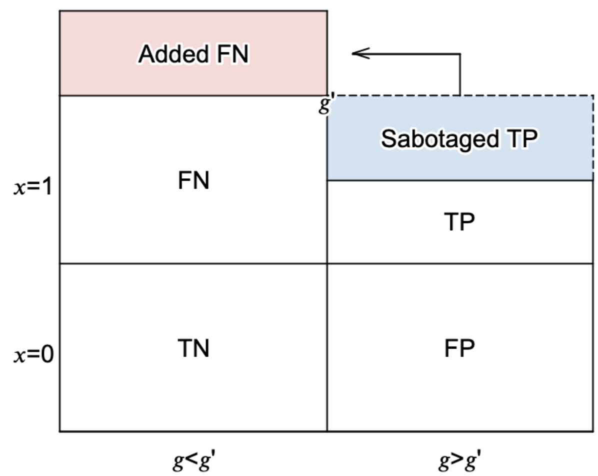
is large enough. This ensures that the reform is a better choice than the status quo in expectation, given $g \geq g'$.

On the other hand, if the voter is too stringent so that g' is too high, he risks not choosing the reform when it is better than the status quo. So, he wants to pick a low enough g' such that the false omission rate (FOR), i.e.

$$\Pr[\omega = 1 | g < g'] = \frac{\Pr[FN]}{\Pr[TN] + \Pr[FN]}$$

is small. This ensures that the reform is expected to perform worse than the status quo given $g < g'$. Evidently, at the cutoff g', the voter is indifferent between the risk of true positives and false negatives.

Figure A1: The Effect of Resistance on Voter Learning



The blue shaded area "resistanced TP" illustrates the PPV effect. The red shaded area "Added FN" illustrates the FOR effect.

Now, consider the additional obfuscation through bureaucratic resistance. Assume bureaucrats resist reform that would otherwise be successful and supported by voters (i.e., $\omega = 1$ and $g > g'$). Hence, with resistance, some of the true positives turn into false negatives with probability $(1 - \kappa')$. This change has two countervailing effects. Figure A1 provides the intuition for this result. Firstly, it *decreases* $\Pr[\omega = 1|g \geq g']$ by lowering $\Pr[TP]$ (the blue shaded area "resistanced TP"). Intuitively, knowing that resistance lowers the likelihood that the voter observes $g > g'$ when it is indeed valuable (i.e. when $\omega = 1$), the voter is inclined to attribute a high $g > g'$ to mere luck rather than its actual value (i.e., a false positive). Formally, for the probability of resistance $1 - \kappa'$,

$$\Pr[\omega = 1|g \geq g'] = \frac{\kappa' \Pr[TP]}{\kappa' \Pr[TP] + \Pr[FP]} < \frac{\Pr[TP]}{\Pr[TP] + \Pr[FP]}.$$

We call this the *PPV effect.*

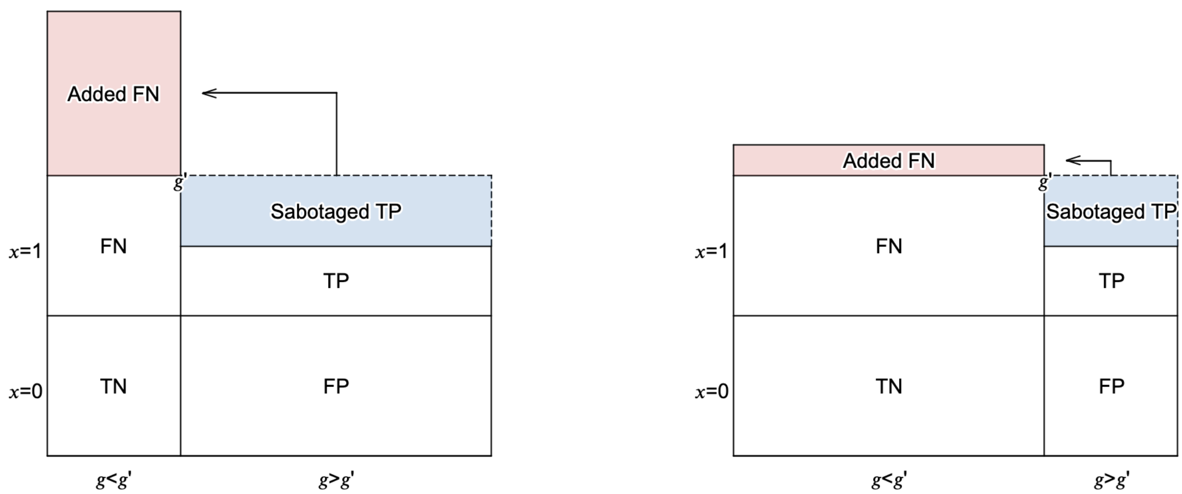
Secondly, the change from TP to FN *increases* $\Pr[\omega = 1|g < g']$ by increasing $\Pr[FN]$ (the red shaded area "Added FN"). Namely, when the voter takes into account the fact that some of the negative signals that he observes are due to resistance, his evaluation of the reform given a negative signal will increase as resistance becomes more likely. That is,

$$\Pr[\omega = 1|g < g'] = \frac{\Pr[FN] + (1 - \kappa') \Pr[TP]}{\Pr[FN] + (1 - \kappa') \Pr[TP] + \Pr[TN]} > \frac{\Pr[FN]}{\Pr[FN] + \Pr[TN]}.$$

We call this the *FOR effect.*

Which effect dominates depends on the initial level of g'. See Figure A2 for an illustration. If g' is high enough so that $g \geq g'$ is rare, the voter is more worried about false positives than false negatives—the FOR effect is low and dominated by the PPV effect.[7] In contrast, if g' is low, the voter faces higher risks of false negatives—the FOR effect is more likely to dominate the PPV effect.[8] Taken together, the effect of resistance on voter behavior depends on what type of wrong inference the voter is most worried about. If the PPV effect dominates the FOR effect, the voter is better off being more stringent and choosing a higher g'. In contrast, if the FOR effect dominates the PPV effect, the voter is better off being more lenient and choosing a lower g'.

Figure A2: Resistance's Effects on Voter Inference Conditional on g'



(a) When g' is low: FOR dominates PPV; resistance decreases g'

(b) When g' is high: PPV dominates FOR; resistance increases g'

It is noteworthy that this result depends on the assumption that bureaucrats can only change TP into FN by sabotaging the reform. For instance, even if bureaucrats do not know ω when they make their decision on resistance, as long as resistance can affect g's distribution only when the reform actually works, the logic above holds.

D.2 Example of Learning Effects

Here, we provide a specific example for the results discussed in Section D.1, fixing the values of g' to those shown in Figure A2. The area of each cell represents the probability of each event and adds up to one. In both panels, the ex-ante total probability of successful reform $\Pr[\omega = 1] = \Pr[TP] + \Pr[FN] = 1/2$. Without resistance,

$$\Pr[\omega = 1 | g \geq g'] = \Pr[\omega = 1 | g < g'] = \frac{1}{2}.$$

[7]We provide calculations of these quantities based on Figure A2 in the next section.

[8]The logic above is similar to that of the main results in Heo and Landa (2024). For further formal discussion on the decision problems with a stochastic process, see Patty and Penn (2023).

If bureaucrats resist, they do so with probability 1/2, and TP (blue shaded area in broken lines, "resistanced TP") becomes FN (red shaded area in solid lines, "Added FN").

In Panel (a), the voter's cutoff is high ($g' = 0.7$), so observing a high signal is rare ($\Pr[g \geq g'] = 0.3$). As resistance decreases $\Pr[TP]$ by 50%,

$$\Pr[\omega = 1|g \geq g'] = \frac{\Pr[TP]}{\Pr[TP] + \Pr[FP]} = \frac{0.3 * 0.5 * 0.5}{0.3 * 0.5 * 0.5 + 0.3 * 0.5} = \frac{1}{3} < \frac{1}{2},$$

and

$$\Pr[\omega = 1|g < g'] = \frac{\Pr[FN]}{\Pr[FN] + \Pr[TN]} = \frac{0.7 * 0.5 + 0.3 * 0.5 * 0.5}{0.7 * 0.5 + 0.3 * 0.5 * 0.5 + 0.7 * 0.5} = \frac{0.85}{1.55} \approx 0.548 > \frac{1}{2}.$$

Evidently, the PPV effect is larger than the FOR effect.

In Panel (b), the voter's cutoff is low ($g' = 0.3$), so a positive signal is relatively more prevalent ($\Pr[g \geq g'] = 0.7$). Without resistance,

$$\Pr[\omega = 1|g \geq g'] = \Pr[\omega = 1|g < g'] = \frac{1}{2}.$$

As resistance decreases $\Pr[TP]$ by 50%,

$$\Pr[\omega = 1|g \geq g'] = \frac{\Pr[TP]}{\Pr[TP] + \Pr[FP]} = \frac{0.7 * 0.5 * 0.5}{0.7 * 0.5 * 0.5 + 0.7 * 0.5} = \frac{1}{3} < \frac{1}{2},$$

and

$$\Pr[\omega = 1|g < g'] = \frac{\Pr[FN]}{\Pr[FN] + \Pr[TN]} = \frac{0.3 * 0.5 + 0.7 * 0.5 * 0.5}{0.3 * 0.5 + 0.7 * 0.5 * 0.5 + 0.3 * 0.5} = \frac{0.65}{0.95} \approx 0.684 > \frac{1}{2}.$$

Here, the FOR effect is larger and dominates the PPV effect. For the general result, see the Appendix of Heo and Landa (2024).

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